Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Tuesday 20 October 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **9FM0/3C**

Further Mathematics

Advanced

Paper 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1/1/1/1/

P66507A0128

1. A particle P of mass 0.5 kg is moving with velocity $(4\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ when it receives an impulse $\mathbf{J}$ N s. Immediately after receiving the impulse, P is moving with velocity $(-\mathbf{i} + 6\mathbf{j})\,\text{m s}^{-1}$.

(a) Find the magnitude of $\mathbf{J}$.

(4)

The angle between the direction of the impulse and the direction of motion of P immediately before receiving the impulse is $\alpha°$

(b) Find the value of α

(3)

(a) Use impulse-momentum principle with vectors

$$I = m(v-u)$$

$$J = 0.5\left(\begin{pmatrix}-1\\6\end{pmatrix} - \begin{pmatrix}4\\3\end{pmatrix}\right) \quad \text{M1A1}$$

velocity after   velocity before

$$J = 0.5\begin{pmatrix}-5\\3\end{pmatrix} = \begin{pmatrix}-\frac{5}{2}\\ \frac{3}{2}\end{pmatrix}$$ J as a vector

Use Pythagoras' Theorem to get magnitude:

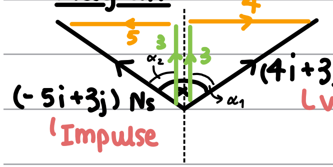
$$\sqrt{(-2.5)^2 + (1.5)^2} \quad \text{M1}$$

$$\text{A1} \quad = \frac{\sqrt{34}}{2}\ \text{Ns}$$ magnitude of Impulse

units for Impulse, Newton-Seconds

(b) Method 1 – trigonometry

Diagram



$$\alpha = \alpha_1 + \alpha_2$$

$$\tan\alpha_1 = \frac{4}{3} \qquad \alpha_1 = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\tan\alpha_2 = \frac{5}{3} \qquad \alpha_2 = \tan^{-1}\left(\frac{5}{3}\right)$$

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{5}{3}\right) \quad \text{M1A1}$$

$$\alpha = 112° \text{ to 3sf} \quad \text{A1}$$

Method 2 – scalar product formula

$$\cos\alpha = \frac{\begin{pmatrix}4\\3\end{pmatrix}\cdot\begin{pmatrix}-5\\3\end{pmatrix}}{\sqrt{25}\times\sqrt{34}} \quad \text{M1A1}$$

$$\cos\alpha = \frac{-20+9}{5\sqrt{34}}$$

$$\alpha = \cos^{-1}\left(\frac{-20+9}{5\sqrt{34}}\right)$$

$$\alpha = 112° \text{ to 3sf} \quad \text{A1}$$

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Question 1 continued

(Total for Question 1 is 7 marks)

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2. A truck of mass 1200 kg is moving along a straight horizontal road.

At the instant when the speed of the truck is $v \text{ m s}^{-1}$, the resistance to the motion of the truck is modelled as a force of magnitude $(900 + 9v)$ N.

The engine of the truck is working at a constant rate of 25 kW.

(a) Find the deceleration of the truck at the instant when $v = 25$

(4)

Later on, the truck is moving up a straight road that is inclined at an angle θ to the horizontal, where $\sin\theta = \dfrac{1}{20}$

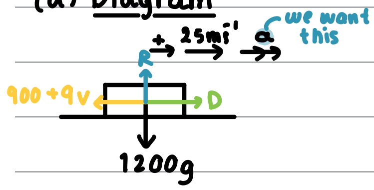
At the instant when the speed of the truck is $v \text{ m s}^{-1}$, the resistance to the motion of the truck from non-gravitational forces is modelled as a force of magnitude $(900 + 9v)$ N.

When the engine of the truck is working at a constant rate of 25 kW the truck is moving up the road at a constant speed of $V \text{ m s}^{-1}$.

(b) Find the value of V.

(5)

(a) Diagram



We are looking for acceleration $\therefore$ use $\Sigma F_x = ma$

$D - (900 + 9(25)) = 1200a$ M1

To get D we will use Power.

Formula for Power:

Power (W) $P = Dv$ Driving force (N), velocity (ms⁻¹)

$P = 25\text{kW}$ – ×1000 → 25000W
$D = D$
$v = 25\text{ms}^{-1}$

Substitute: $25\,000 = 25D$ M1

$D = 1000\text{N}$

Substitute D back and solve for a:

$1000 - 1125 = 1200a$ A1

$\frac{-125}{1200} = a$

$a = -\frac{5}{48} = 0.104\text{ms}^{-2}$ to 3sf A1

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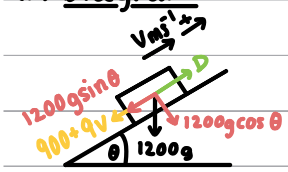
Question 2 continued

(b) Diagram



Since the speed is constant, use $\Sigma F_x = 0$:

$D = 900 + 9V + 1200g\sin\theta$ M1A1A1

To get D we will use Power.

Formula for Power:

Power (W) ⟶ $P = Dv$

Driving force (N) — Velocity (ms^{-1})

$P = 25kW \xrightarrow{\times 1000} 25\,000W$

$D = D$

$V = V$

Substiture:

$25\,000 = Dv$

$\frac{25\,000}{V} = D$

Substitute D back and solve for V:

$\frac{25\,000}{V} = 900 + 9V + \frac{\cancel{1200}^{60}g}{\cancel{20}}$

$25\,000 = 900V + 9V^2 + 60gV$

$25\,000 = 1488V + 9V^2$

$0 = 9V^2 + 1488V - 25\,000$ M1

↓ Use Quadratic Formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$V = 15.4ms^{-1}$ to 3sf A1

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3. Two particles, A and B, have masses $3m$ and $4m$ respectively. The particles are moving in the same direction along the same straight line on a smooth horizontal surface when they collide directly. Immediately before the collision the speed of A is $2u$ and the speed of B is u.

The coefficient of restitution between A and B is e.

(a) Show that the direction of motion of each of the particles is unchanged by the collision.

(8)

After the collision with A, particle B collides directly with a third particle, C, of mass $2m$, which is at rest on the surface.

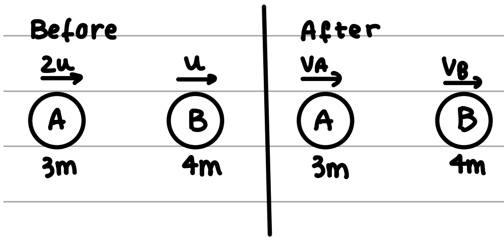
The coefficient of restitution between B and C is also e.

(b) Show that there will be a second collision between A and B.

(6)

Question 3 continued

(a) Diagram



We can use the conservation of linear momentum to get an equation

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

initial velocity   final velocity

Substitute: $3\cancel{m}(2u) + 4\cancel{m}(u) = 3\cancel{m}v_A + 4\cancel{m}v_B$

$6u + 4u = 3v_A + 4v_B$

$10u = 3v_A + 4v_B$ Eq.1   M1A1

We can use Newton's Law of Restitution to get an equation.

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$

coefficient of restitution   initial speed   final speed

Substitute: $e(2u - u) = v_B - v_A$

$eu = v_B - v_A$ Eq. 2   M1A1

Solve simultaneously Eq1 and Eq2: M1

Get v_B: $10u = 3v_A + 4v_B$ use elimination method

$eu = v_B - v_A$ $|\times 3|$ $3eu = -3v_A + 3v_B$ +

$10u + 3eu = 7v_B$

$v_B = \frac{u(10+3e)}{7}$ A1

Now get v_A: $10u = 3v_A + 4v_B$

$eu = v_B - v_A$ $|\times -4|$ $-4eu = 4v_A - 4v_B$ +

$10u - 4eu = 7v_A$

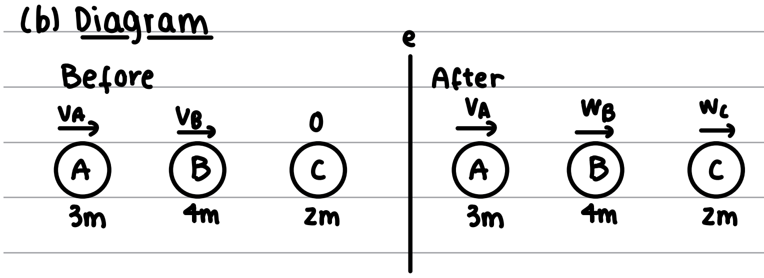
$v_A = \frac{2u(5-2e)}{7}$ A1

We know that $0 \le e \le 1$, $\therefore$ we get that $10 + 3e > 0 \therefore v_B > 0$ and $5 - 2e > 0 \therefore v_A > 0$.

Hence we know that both particles move in the original direction. A1

Question 3 continued

(b) Diagram



V_A and V_B from (a):

$V_A = \frac{2u}{7}(5-2e)$

$V_B = \frac{u}{7}(10+3e)$

We need W_B.

Consider only B and C for the collision between them.

We can use the conservation of linear momentum to get this.

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A \underbrace{u_A}_{\text{initial velocity}} + m_B u_B = m_A \underbrace{v_A}_{\text{final velocity}} + m_B v_B$

Substitute: $4\cancel{m}\left(\frac{u}{7}(10+3e)\right) + \overset{0}{\cancel{2m(0)}} = 4\cancel{m}W_B + 2\cancel{m}w_C$

$\frac{4u}{7}(10+3e) = 4w_B + 2w_C$ Eq1 M1

We can use Newton's Law of Restitution to get an equation.

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $\underbrace{e}_{\text{coefficient of restitution}}\underbrace{(u_A - u_B)}_{\text{initial speed}} = \underbrace{v_B - v_A}_{\text{final speed}}$

Substitute: $e\left(\frac{u}{7}(10+3e) - 0\right) = w_C - w_B$

$\frac{ue}{7}(10+3e) = w_C - w_B$ Eq2 M1

Solve simultaneously Eq1 and Eq2 to get w_B:

$\frac{4u}{7}(10+3e) = 4w_B + 2w_C$ use elimination method

$\frac{ue}{7}(10+3e) = w_C - w_B$ $|\times -2|$ $-\frac{2eu}{7}(10+3e) = 2w_B - 2w_C$ +

$\frac{4u}{7}(10+3e) - \frac{2eu}{7}(10+3e) = 6w_B$

$(10+3e)\left(\frac{4u}{7} - \frac{2eu}{7}\right) = 6w_B$

$\frac{2u}{7}(10+3e)(2-e) = 6w_B$

$\frac{2u}{42}(20-4e-3e^2) = w_B$

$\frac{u}{21}(20-4e-3e^2) = w_B$ M1

Now we will look at the difference between V_A and W_B:

$V_A - W_B = \frac{u}{7}(10-4e) - \frac{u}{21}(20-4e-3e^2)$

$= \frac{3}{21}u(10-4e) - \frac{u}{21}(20-4e-3e^2)$

$= \frac{u}{21}(30-12e-20+4e+3e^2)$

$= \frac{u}{21}(3e^2-8e+10)$ difference between v_A and v_B

M1

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Question 3 continued

This difference between v_A and w_B should be positive (v_A should be larger than w_B, $\therefore v_A - w_B > 0$).

$$v_A - w_B > 0 \quad \text{M1}$$

$$\frac{u}{21}(3e^2 - 8e + 10) > 0$$

$$3e^2 - 8e + 10 > 0$$

complete square

we need to show that this is always positive

$$3\left(e - \frac{4}{3}\right)^2 + \frac{14}{3}$$

As $\left(e - \frac{4}{3}\right)^2 > 0$ for all values of e and $\frac{14}{3} > 0$, $3e^2 - 8e + 10 > 0$ for all values of e, $\therefore v_A - w_B > 0$ for all e.

Hence, we have shown that there is a 2^{nd} collision between A and B. A1

(Total for Question 3 is 14 marks)

4. [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

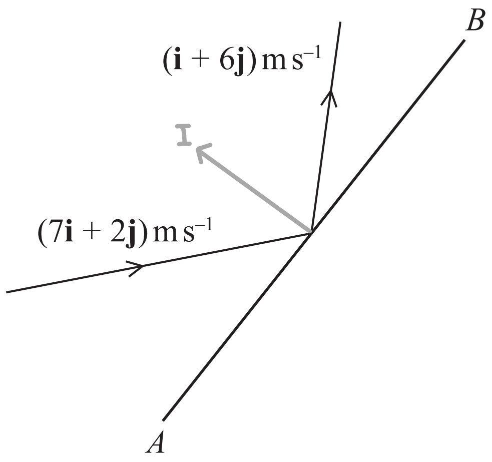


Figure 1

Figure 1 represents the plan view of part of a smooth horizontal floor, where AB represents a fixed smooth vertical wall.

A small ball of mass 0.5 kg is moving on the floor when it strikes the wall.

Immediately before the impact the velocity of the ball is $(7\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$.

Immediately after the impact the velocity of the ball is $(\mathbf{i} + 6\mathbf{j})\,\mathrm{m\,s^{-1}}$.

The coefficient of restitution between the ball and the wall is e.

(a) Show that AB is parallel to $(2\mathbf{i} + 3\mathbf{j})$.

(4)

(b) Find the value of e.

(5)

(a) We know that the impulse is perpendicular to the wall. We will use this fact to show this.

Impulse is the change in momentum

Formula for change in momentum:

$$\Delta\text{momentum} = \overset{\text{mass}}{m}v_{\text{final}} - m\underset{\text{velocity}}{v_{\text{initial}}}$$

Substitute: $I = m(v-u)$ M1

$$I = 0.5\left(\begin{pmatrix}1\\6\end{pmatrix} - \begin{pmatrix}7\\2\end{pmatrix}\right)$$

$$= 0.5 \times \begin{pmatrix}-6\\4\end{pmatrix} = \begin{pmatrix}-3\\2\end{pmatrix}$$ Impulse A1

We need to use the scalar product to show that $\begin{pmatrix}-3\\2\end{pmatrix}$ and $\begin{pmatrix}2\\3\end{pmatrix}$ are perpendicular

$$\begin{pmatrix}-3\\2\end{pmatrix}\cdot\begin{pmatrix}2\\3\end{pmatrix}$$ scalar product: $\begin{pmatrix}a\\b\end{pmatrix}\cdot\begin{pmatrix}c\\d\end{pmatrix} = ac+bd$

$$= -3(2) + 2(3)$$ M1

$$= -6 + 6 = 0$$

As the scalar product is 0, we have shown that the two vectors are perpendicular and as the Impulse is perpendicular to the wall, AB must be parallel to $\begin{pmatrix}2\\3\end{pmatrix}$. A1

Question 4 continued

(b) Formulae for vector collisions:

$\underset{\sim}{u} \cdot \underset{\sim}{w} = \underset{\sim}{v} \cdot \underset{\sim}{w}$ → parallel vector to wall

initial speed   final speed

$-e\,\underset{\sim}{u} \cdot \underset{\sim}{I} = \underset{\sim}{v} \cdot \underset{\sim}{I}$ → perpendicular vector to wall

Let's write down everything we know:

$\underset{\sim}{u} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$ $\underset{\sim}{v} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$

$\underset{\sim}{w} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\underset{\sim}{I} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ from (a) M1

$e = e$ ← we've looking for this

We will use the 2nd formula to get e:

Scalar Product: $\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} c \\ d \end{pmatrix} = ac + bd$

Substitute:

$$-e \times \begin{pmatrix} 7 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$ M1A1A1

$$-e(-21+4) = -3+12$$

$$-e \times -17 = 9$$

$$e = \frac{9}{17}$$ value of e

A1

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. A smooth uniform sphere P has mass 0.3 kg. Another smooth uniform sphere Q, with the same radius as P, has mass 0.2 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision the velocity of P is $(4\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$ and the velocity of Q is $(-3\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$.

At the instant of collision, the line joining the centres of the spheres is parallel to $\mathbf{i}$.

The kinetic energy of Q immediately after the collision is half the kinetic energy of Q immediately before the collision.

(a) Find

(i) the velocity of P immediately after the collision, v_P

(ii) the velocity of Q immediately after the collision, v_Q

(iii) the coefficient of restitution between P and Q, e

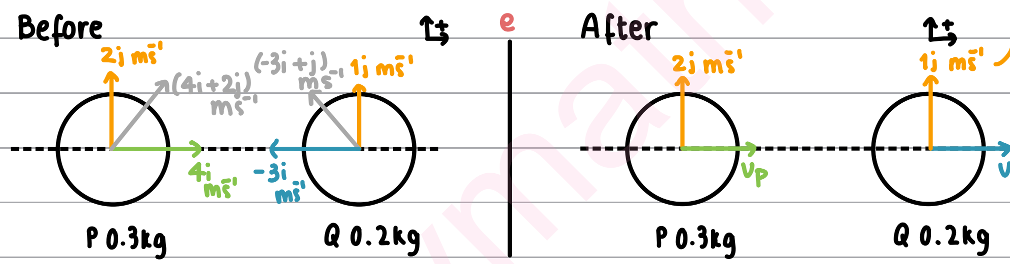
carefully justifying your answers.

(11)

(b) Find the size of the angle through which the direction of motion of P is deflected by the collision.

(3)

(a) Diagram



the components perpendicular to the line of centers stay the same!

B1

First, consider the KE of Q:

Formula for Kinetic Energy:

$$KE = \frac{1}{2}mv^2$$

mass, velocity

We are given that: $KE_F = \frac{1}{2}KE_I$

Substitute:

final speed of Q; Pythagoras' Theorem

$$\frac{1}{2}(0.2)\left(\sqrt{v_Q^2+1^2}\right)^2 = \frac{1}{2}\times\frac{1}{2}(0.2)\left(\sqrt{(-3)^2+1^2}\right)^2$$ M1 A1

$$0.1\times(v_Q^2+1) = 0.05(9+1)$$

$$0.1(v_Q^2+1) = 0.5$$

$$v_Q^2+1 = 5$$

$$v_Q^2 = 4$$

$$v_Q = \pm 2$$

Question 5 continued

Parallel to the line of centers:

We can use the conservation of linear momentum to get an equation.

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$ (u_A: initial velocity, v_A: final velocity)

Substitute:

$$0.3(4) + 0.2(-3) = 0.3v_P + 0.2v_Q$$

$$1.2 - 0.6 = 0.3v_P + 0.2v_Q$$ multiply both sides by 10 so we don't have decimals.

M1A1 Eq.1 $6 = 3v_P + 2v_Q$

We can use Newton's Law of Restitution to get an equation.

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$ (e: coefficient of restitution, u: initial speed, v: final speed)

Substitute:

$$e(4-(-3)) = v_Q - v_P$$

M1A1 $7e = v_Q - v_P$ Eq. 2

Solve simultaneously Eq1 and Eq2:

i. to get v_P

$$6 = 3v_P + 2v_Q$$ use elimination method

$$7e = v_Q - v_P \quad |\times -2| \quad -14e = 2v_P - 2v_Q \quad +$$

$$6 - 14e = 5v_P$$

$$v_P = \frac{6-14e}{5}$$

ii. to get v_Q

$$6 = 3v_P + 2v_Q$$ use elimination method

$$7e = v_Q - v_P \quad |\times 3| \quad 21e = -3v_P + 3v_Q \quad +$$

$$6 + 21e = 5v_Q$$ and we know $v_Q = \pm 2$

If $v_Q = -2$:

$$6 + 21e = 5(-2)$$

$$6 + 21e = -10$$

which gives $e = \frac{-16}{21}$. this is NOT possible since $0 \le e \le 1$. A1

$\therefore$ If $v_Q = 2$:

$$6 + 21e = 5(2)$$

$$21e = 10 - 6$$

iii. $e = \frac{4}{21}$ value of e M1

Question 5 continued

$\therefore$ Now we can get V_P, V_Q and the final velocities using our value for e:

$$V_P = \frac{6-\cancel{14}^{\,2}\left(\frac{4}{\cancel{21}_{\,3}}\right)}{5}$$

$$= \frac{6-\frac{8}{3}}{5}$$

$$= \frac{\frac{10}{3}}{5}$$

$$V_P = \frac{2}{3}$$

$$V_Q = 2$$

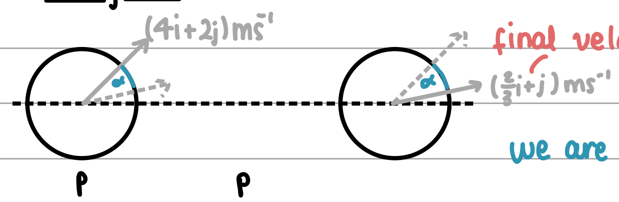
$\therefore$ For:

i. P the final velocity is $\left(\frac{2}{3}i + 2j\right)ms^{-1}$

ii. Q the final velocity is $(2i + j)ms^{-1}$ A1

iii. the value of e is $\frac{4}{21}$ A1

(b) Diagram



final velocity from (a)

we are looking for angle α

Let's use the scalar product formula to get the angle.

$$\cos\theta = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}| \times |\underset{\sim}{b}|}$$

Substitute in our values:

$$\cos\alpha = \frac{\begin{pmatrix}4\\2\end{pmatrix} \cdot \begin{pmatrix}\frac{2}{3}\\2\end{pmatrix}}{\sqrt{4^2+2^2} \times \sqrt{2^2+\left(-\frac{2}{3}\right)^2}}$$ M1 A1

$$\cos\alpha = \frac{4+\frac{8}{3}}{\sqrt{20} \times \sqrt{4\frac{4}{9}}}$$

$$\alpha = \cos^{-1}\left(\frac{\frac{20}{3}}{\frac{20\sqrt{2}}{3}}\right)$$

$$\alpha = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$\alpha = 45°$ angle deflected

A1

Question 5 continued

(Total for Question 5 is 14 marks)

6. A light elastic string with natural length l and modulus of elasticity kmg has one end attached to a fixed point A on a rough inclined plane. The other end of the string is attached to a package of mass m.

The plane is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$

The package is initially held at A. The package is then projected with speed $\sqrt{6gl}$ up a line of greatest slope of the plane and first comes to rest at the point B, where $AB = 3l$.

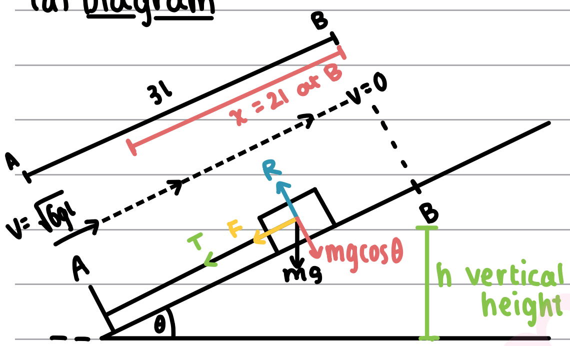
The coefficient of friction between the package and the plane is $\frac{1}{4}$

By modelling the package as a particle,

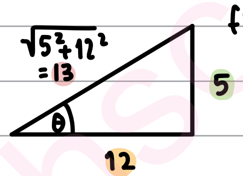
(a) show that $k = \frac{15}{26}$ in λ **(6)**

(b) find the acceleration of the package at the instant it starts to move back down the plane from the point B. **(5)**

(a) Diagram



GPE = 0 at A, as a reference point



from $\tan\theta = \frac{5}{12}$

$\therefore \cos\theta = \frac{12}{13}$

$\sin\theta = \frac{5}{13}$

★ Work-Energy Principle: an increase of KE/GPE/EPE is caused by an equal amount of positive work done on the body (e.g. engine) and a decrease of KE/GPE/EPE is caused by an equal amount of negative work done on the body (e.g. friction).

★ Remember the work-energy formulae:

Either: WD by force + KE_i + GPE_i + EPE_i = KE_f + GPE_f + EPE_f + WD against friction

work done, initial kinetic, initial grav. potential, initial elastic potential, final kinetic, final grav. potential, final elastic potential, work lost to friction

OR: WD by force + KE_i + GPE_i + EPE_i − WD by friction = KE_f + GPE_f + EPE_f

work done, initial kinetic, initial grav. potential, initial elastic potential, we subtract this since it leaves the system as heat!, final kinetic, final grav. potential, final elastic potential

★ Formulae for KE, GPE and EPE:

$KE = \frac{1}{2}mv^2$ — velocity, mass

$GPE = mgh$ — change in height, mass, $g = 9.8\,ms^{-2}$

$EPE = \frac{\lambda x^2}{2l}$ — modulus of elasticity, extension of string/spring, natural length of the string/spring

P 6 6 5 0 7 A 0 2 0 2 8

Question 6 continued

Let's substitute what we know:

$$\frac{1}{2}m(\sqrt{6gl})^2 + \cancel{mg(0)} + \cancel{\frac{\lambda(0)^2}{2l}} - 3lF = \cancel{\frac{1}{2}m(0)^2} + mgh + \frac{\lambda(2l)^2}{2l}$$ M1A1

∴ We need to find F and h.

I. For F

Since there is motion, we know F is maximum and $F_{max} = \mu R$

To get R use $\Sigma F_y = 0$ vertically:

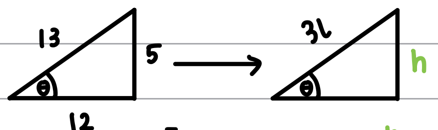
$$R = mg\cos\theta = \frac{12}{13}mg$$

∴ get F:

$$F = \frac{1}{\cancel{4}} \times \frac{\cancel{12}^{\,3}}{13}mg$$

$$F = \frac{3}{13}mg$$

II. For h



$\sin\theta = \frac{5}{13} \longrightarrow \sin\theta = \frac{h}{3l}$

$$\therefore \frac{5}{13} = \frac{h}{3l}$$

$$h = \frac{15}{13}l$$

Substitute back into our WE equation: F, h, we want this!

$$\frac{1}{2}m(\sqrt{6gl})^2 - 3l\left(\frac{3}{13}mg\right) = mg\left(\frac{15}{13}l\right) + \frac{kmg(2l)^2}{2l}$$ B1B1B1

Solve for k:

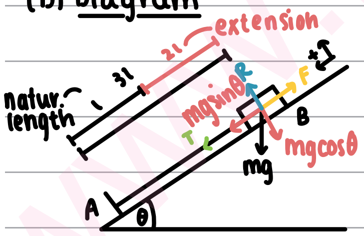
$$\frac{1}{2}m(6gl) - \frac{9}{13}mgl = \frac{15}{13}mgl + \frac{{}^{2}\cancel{4}l^2kmg}{\cancel{2l}}$$

$$3\cancel{mgl} - \frac{9}{13}\cancel{mgl} - \frac{15}{13}\cancel{mgl} = 2k\cancel{mgl}$$ cancel mgl

$$\frac{15}{13} = 2k$$

$$k = \frac{15}{26}$$ hence shown A1

(b) Diagram



At the point of slipping, F is max., ∴ $F = \mu R$

$F = \frac{3}{13}mg$ (from (a))

Formula for tension in the string:

$$T = \frac{\lambda x}{l}$$

Substitute:

$$T = \frac{\frac{15}{26}mg \times 2\cancel{l}}{\cancel{l}}$$ B1

$$T = \frac{15}{26}mg$$

Now we can use $\Sigma F_x = ma$ to get acceleration

$$T + mg\sin\theta - F = m\alpha$$ we want a M1A1

$$\frac{15}{26}\cancel{m}g + \frac{5}{13}\cancel{m}g - \frac{3}{13}\cancel{m}g = \cancel{m}\alpha$$ cancel m's A1

$$\frac{17}{13}g = \alpha$$ acceleration at b. A1

Question 6 continued

Question 6 continued

(Total for Question 6 is 11 marks)

7.

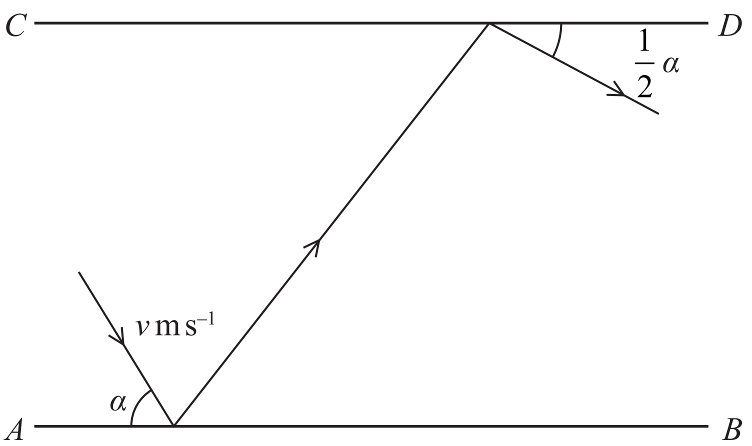


Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where AB and CD represent fixed vertical walls, with AB parallel to CD.

A small ball is projected along the floor towards wall AB. Immediately before hitting wall AB, the ball is moving with speed $v\text{ m s}^{-1}$ at an angle α to AB, where $0 < \alpha < \frac{\pi}{2}$

The ball hits wall AB and then hits wall CD.

After the impact with wall CD, the ball is moving at angle $\frac{1}{2}\alpha$ to CD.

The coefficient of restitution between the ball and wall AB is $\frac{2}{3}$

The coefficient of restitution between the ball and wall CD is also $\frac{2}{3}$

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

(a) Show that $\tan\left(\frac{1}{2}\alpha\right) = \frac{1}{3}$

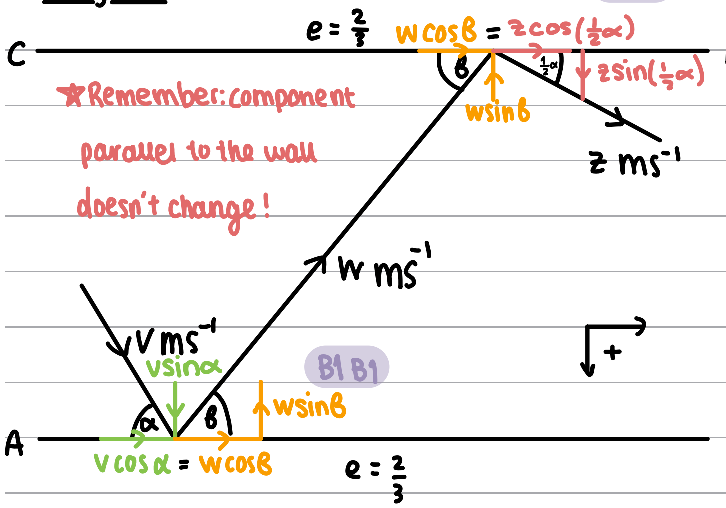
(7)

(b) Find the percentage of the initial kinetic energy of the ball that is lost as a result of the two impacts.

(4)

Question 7 continued

(a) Diagram B1 B1



Perpendicular to the walls, use Impact law:

$e v\sin\alpha = -w\sin\beta$

$\frac{2}{3}v\sin\alpha = -w\sin\beta$ AB

$-e w\sin\beta = z\sin(\frac{1}{2}\alpha)$

$-\frac{2}{3}w\sin\beta = z\sin(\frac{1}{2}\alpha)$ CD

$\therefore \frac{2}{3}\times\frac{2}{3}v\sin\alpha = z\sin(\frac{1}{2}\alpha)$ (I)

Parallel to the walls:

$v\cos\alpha = w\cos\beta$

$w\cos\beta = z\cos(\frac{1}{2}\alpha)$

$\therefore v\cos\alpha = z\cos(\frac{1}{2}\alpha)$ (II)

Hence, by dividing (I) and (II)

$$\frac{(I)}{(II)} \Rightarrow \frac{\frac{4}{9}\cancel{v}\sin\alpha = \cancel{z}\sin(\frac{1}{2}\alpha)}{\cancel{v}\cos\alpha = \cancel{z}\cos(\frac{1}{2}\alpha)}$$ cancel v's and z's

$$\Rightarrow \frac{4}{9}\tan\alpha = \tan(\frac{1}{2}\alpha)$$ M1 apply $\tan\theta = \frac{\sin\theta}{\cos\theta}$ identity

We can use the double angle formula for $\tan\alpha$:

$$\tan\alpha = \frac{2(\tan\frac{\alpha}{2})}{1-\tan^2(\frac{\alpha}{2})}$$

Let's set $\tan\frac{\alpha}{2} = t$, $\therefore$ $t = \frac{4}{9}\tan\alpha$ use double-angle formula

from $\Rightarrow \frac{4}{9}\tan\alpha = \tan(\frac{1}{2}\alpha)$ above.

$$t = \frac{4}{9}\tan\alpha = \frac{4}{9}\left(\frac{2t}{1-t^2}\right)$$ since $t = \tan\frac{\alpha}{2}$

M1

$$t = \frac{4\times 2t}{9(1-t^2)}$$

Solve for t:

$$9(1-t^2) = \frac{8\cancel{t}}{\cancel{t}}$$

$$1-t^2 = \frac{8}{9}$$

$$t^2 = \frac{1}{9}$$

$$t = \frac{1}{3}$$ A1

Since $t = \tan\frac{\alpha}{2}$ and $t = \frac{1}{3}$, $\tan\frac{\alpha}{2} = \frac{1}{3}$ has been shown.

Question 7 continued

(b) Formula for %KE lost:

$$\%KE = 100 \times \left(1 - \frac{KE_F}{KE_I}\right)$$

Formula for Kinetic Energy:

$$KE = \frac{1}{2} \underbrace{m}_{\text{mass}} \underbrace{v^2}_{\text{velocity}}$$

To get KE lost: $\Delta KE = KE_F - KE_I$ M1

$KE_I = \frac{1}{2}mv^2$

$KE_F = \frac{1}{2}mz^2$

We need the final speed, z.

To get this, let's square and add (I) and (II) from (a)

$$\frac{4}{9}v\sin\alpha = z\sin\left(\tfrac{1}{2}\alpha\right) \quad (I)$$

$$v\cos\alpha = z\cos\left(\tfrac{1}{2}\alpha\right) \quad (II)$$

$$(I)^2 + (II)^2 = \frac{16}{81}v^2\sin^2\alpha + v^2\cos^2\alpha = z^2\sin^2\left(\tfrac{1}{2}\alpha\right) + z^2\cos^2\left(\tfrac{1}{2}\alpha\right)$$ use identity

$$v^2\left(\frac{16}{81}\sin^2\alpha + \cos^2\alpha\right) = z^2\left(\underbrace{\sin^2\left(\tfrac{1}{2}\alpha\right) + \cos^2\left(\tfrac{1}{2}\alpha\right)}_{=1}\right)$$ $\sin^2\alpha + \cos^2\alpha \equiv 1$

$$v^2\left(\frac{16}{81}\sin^2\alpha + \cos^2\alpha\right) = z^2$$

Substitute our z^2 into %KE lost formula

cancel $\frac{1}{2}mv^2$

$$\%KE \text{ lost} = 100 \times \left(1 - \frac{\cancel{\frac{1}{2}mv^2}\left(\frac{16}{81}\sin^2\alpha + \cos^2\alpha\right)}{\cancel{\frac{1}{2}mv^2}}\right)$$ M1

$$= 100 \times \left(1 - \left(\frac{16}{81} \times \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2\right)\right)$$

$$= 100 \times \left(1 - \frac{16}{225} - \frac{16}{25}\right)$$

$= 28.8\%$ of KE was lost A1

☆ to get $\sin\alpha$ and $\cos\alpha$:

$\tan\alpha = \frac{9}{4}\tan\frac{\alpha}{2}$

$= \frac{9}{4} \times \frac{1}{3} = \frac{3}{4} = \tan\alpha$ B1

$\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$

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Question 7 continued

Turn over

Question 7 continued

(Total for Question 7 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS